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HIDING AND CONFINING CHARGES VIA "TUBE-LIKE" WORMHOLES

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We describe two interesting effects in wormhole physics. First, we find that a genuinely charged matter source of gravity and electromagnetism may appear *electrically neutral* to an external observer — a phenomenon opposite to the famous Misner–Wheeler "charge without charge" effect. We show that this phenomenon takes place when coupling a bulk gravity/nonlinear-gauge-field system self-consistently to a codimension-one charged *lightlike* brane as a matter source. The "charge-hiding" effect occurs in a self-consistent wormhole solution of the above coupled gravity/nonlinear-gauge-field/lightlike-brane system which connects a noncompact "universe," comprising the exterior region of Schwarzschild–(anti-)de Sitter (or purely Schwarzschild) black hole beyond the internal (Schwarzschild) horizon, to a Levi-Civita–Bertotti–Robinson-type ("tube-like") "universe" with two compactified dimensions via a wormhole "throat" occupied by the charged lightlike brane. In this solution the whole electric flux produced by the charged lightlike brane is expelled into the compactified Levi-Civita–Bertotti–Robinson-type "universe" and, consequently, the brane is detected as neutral by an observer in the Schwarzschild–(anti-)de Sitter "universe." Next, the above "charge-hiding" solution can be further generalized to a truly *charge-confining* wormhole solution when we couple the bulk gravity/nonlinear-gauge-field system self-consistently to *two* separate codimension-one charged *lightlike* branes with equal in magnitude but opposite charges. The latter system possesses a "two-throat" wormhole solution, where the "left-most" and the "right-most" "universes" are two identical copies of the exterior region of the neutral Schwarzschild–de Sitter black hole beyond the Schwarzschild horizon, whereas the "middle" "universe" is of generalized Levi-Civita–Bertotti–Robinson "tube-like" form with geometry $dS_2 \times S^2$ (dS_2 being the two-dimensional de Sitter space). It comprises the finite-extent intermediate region of dS_2 between its two horizons. Both "throats" are occupied by the two oppositely charged lightlike branes and the whole electric flux produced by the latter is confined entirely within the middle finite-extent "tube-like" "universe." A crucial ingredient is the special form of the nonlinear gauge field action,

which contains both the standard Maxwell term as well as a square root of the latter. This theory was previously shown to produce a QCD-like confining dynamics in flat space–time.

1. Introduction

In Ref. 1 't Hooft has proposed a consistent quantum description of linear confinement phenomena in terms of effective nonlinear gauge field actions, where the nonlinear terms play the role of effective "infrared counterterms." In particular, he has argued that the energy density of electrostatic field configurations should be a linear function of the electric displacement field in the infrared region (see especially Eq. (5.10) in Ref. 1), which means that an additional term of the form of square root of the standard Maxwell term should appear in the effective action. The simplest way to realize this idea in Minkowski space–time is by considering the following nonlinear effective gauge field model:[2–7]

$$
S = \int d^4 x \, L(F^2) \,, \quad L(F^2) = -\frac{1}{4} F^2 - \frac{f}{2} \sqrt{-F^2} \,,
$$
$$
F^2 \equiv F_{\mu\nu} F^{\mu\nu} \,, \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \,,
$$

$$(1)$$

with f being a positive coupling constant.

Since the Lagrangian $L(F^2)$ in (1) contains both the usual Maxwell term as well as a nonanalytic function of F^2, it is thus a *nonstandard* form of nonlinear electrodynamics. There are various reasons supporting the natural appearance of the "square-root" Maxwell term in effective gauge field actions besides 't Hooft's arguments in Ref. 1. Originally a purely "square-root" Lagrangian in flat space–time $-\frac{f}{2}\sqrt{F^2}$ (in "magnetic"-dominated form) was proposed by Nielsen and Olesen[8] to describe string dynamics (see also Refs. 9–11). Furthermore, it has been shown in Refs. 2–4 that the square root of the Maxwell term naturally arises (in flat space–time) as a result of spontaneous breakdown of scale symmetry of the original scale-invariant Maxwell theory with f appearing as an integration constant responsible for the latter spontaneous breakdown.

As shown in Refs. 3–7 the flat space–time model (1), when coupled to charged fermions, produces a confining effective potential $V(r) = -\frac{\alpha}{r} + \beta r$ (Coulomb plus linear one) which is of the form of the well-known "Cornell" potential[12–14] in quantum chromodynamics (QCD). Also, for static field configurations the model (1) yields the following electric displacement field $\mathbf{D} = \mathbf{E} - \frac{f}{\sqrt{2}} \frac{\mathbf{E}}{|\mathbf{E}|}$. The pertinent energy density turns out to be (there is *no* contribution from the square-root term in (1)) $\frac{1}{2}\mathbf{E}^2 = \frac{1}{2}|\mathbf{D}|^2 + \frac{f}{\sqrt{2}}|\mathbf{D}| + \frac{1}{4}f^2$, so that it indeed contains a term linear w.r.t. $|\mathbf{D}|$ as argued by 't Hooft.[1] Similar connection between $\mathbf{D}$ and $\mathbf{E}$ has been considered as an

example of a "classical model of confinement" in Ref. 15 and analyzed generalizing the methods developed for the "leading logarithm model" in Ref. 16.

The gauge-field system with a square root of the Maxwell term (1) coupled to gravity (cf. Eq. (2) below) was recently studied in Ref. 17 (see the brief review in the following Sec. 2), where the following interesting new features of the pertinent static spherically symmetric solutions have been found:

(i) appearance of a constant vacuum radial electric field (in addition to the Coulomb one) in charged black holes within Reissner–Nordström–de Sitter-type and/or Reissner–Nordström–*anti*-de Sitter-type space–times, in particular, in electrically neutral black holes with Schwarzschild–de Sitter and/or Schwarzschild–*anti*-de Sitter geometry;

(ii) novel mechanism of *dynamical generation* of cosmological constant through the nonlinear gauge field dynamics due to the "square-root" Maxwell term;

(iii) appearance of a confining-type effective potential in charged test particle dynamics in the above black hole backgrounds.

Further, it is interesting to study possible new effects which can take place in the context of wormhole physics where the wormholes are generated due to the presence of nonlinear gauge fields with confining type dynamics. To this end let us recall that Misner–Wheeler "charge without charge" effect[18] stands out as one of the most interesting physical phenomena produced by wormholes. Misner and Wheeler realized that wormholes connecting two asymptotically flat space–times provide the possibility of existence of electromagnetically nontrivial solutions, where the lines of force of the electric field flow from one universe to the other without a source and giving the impression of being positively charged in one universe and negatively charged in the other universe.

For a detailed exposition of the basics of wormhole physics we refer to Visser's book Ref. 19 (see also Refs. 20 and 21) and some more recent accounts.[22–26]

In a recent note[27] we found the opposite effect in wormhole physics, namely, that a genuinely charged matter source of gravity and electromagnetism may appear *electrically neutral* to an external observer. We showed this phenomenon to take place in the coupled gravity/nonlinear-gauge-field system (2) (without bare cosmological constant) self-consistently interacting with a charged lightlike brane as a matter source (cf. Eq. (43) below). In this case the lightlike brane, which connects as a wormhole "throat" a noncompact "universe" with a compactified "universe," is electrically charged, however all of its flux flows into the compactified ("tube-like") "universe" only. No Coulomb field is produced in the noncompact "universe," therefore, the wormhole hides the charge from an external observer in the latter "universe." This charge-hiding effect is exclusively due to the presence of the "square-root" Maxwell term in the nonlinear gauge field action.

A few remarks about the relevance of lightlike branes within the present context are in order. In our previous papers[28–36] we have provided an explicit reparametrization invariant worldvolume Lagrangian formulation of lightlike *p*-branes

("*LL-branes*" for short) (a brief review is given in Sec. 4) and we have used them to construct various types of wormhole, regular black hole and lightlike braneworld solutions in $D = 4$ or higher-dimensional asymptotically flat or asymptotically (anti-)de Sitter bulk space–times. In particular, in Refs. 35 and 36 we have shown that *LL-branes* can trigger a series of spontaneous compactification-decompactification transitions of space–time regions, e.g. from ordinary compactified ("tube-like") Levi-Civita–Bertotti–Robinson space[37–39] to noncompact Reissner–Nordström or Reissner–Nordström–de Sitter region or *vice versa*. Wormholes with "tube-like" structure and regular black holes with "tube-like" core have been previously obtained within different contexts in Refs. 40–48.

Here an important remark about "Einstein–Rosen bridge" wormhole is in order. The nomenclature of "Einstein–Rosen bridge" in several standard textbooks uses the Kruskal–Szekeres manifold, where the "Einstein–Rosen bridge" geometry becomes *dynamical* (see Ref. 49, p. 839, Fig. 31.6, and Ref. 50, p. 228, Fig. 5.15). The latter notion of "Einstein–Rosen bridge" is *not* equivalent to the original Einstein–Rosen's construction,[51] where the space–time manifold is *static* spherically symmetric consisting of two identical copies of the outer Schwarzschild space–time region ($r > 2\,\mathrm{m}$) glued together along the horizon at $r = 2\,\mathrm{m}$. Namely, the two regions in Kruskal–Szekeres space–time corresponding to the outer Schwarzschild space–time region ($r > 2\,\mathrm{m}$) and labeled (I) and (III) in Ref. 49 are generally *disconnected* and share only a two-sphere (the angular part) as a common border ($U = 0$, $V = 0$ in Kruskal–Szekeres coordinates), whereas in the original Einstein–Rosen "bridge" construction[51] the boundary between the two identical copies of the outer Schwarzschild space–time region ($r > 2\,\mathrm{m}$) is a three-dimensional (lightlike) hypersurface ($r = 2\,\mathrm{m}$). In Refs. 31 and 34 it has been shown that the "Einstein–Rosen bridge" in its original formulation[51] naturally arises as the simplest particular case of static spherically symmetric wormhole solutions produced by lightlike branes as gravitational sources, where the two identical "universes" with Schwarzschild outer-region geometry are glued together by a lightlike brane occupying their common horizon — the wormhole "throat." An understanding of this picture within the framework of Kruskal–Szekeres manifold was subsequently provided in Ref. 52, which involves Rindler's elliptic identification of the two antipodal future event horizons.

Let us recall that *LL-branes* by themselves play an important role in modern general relativity. They are singular null (lightlike) hypersurfaces in Riemannian space–time which provide dynamical description of various physically important phenomena in cosmology and astrophysics such as: (i) impulsive lightlike signals arising in cataclysmic astrophysical events;[53] (ii) the "membrane paradigm"[54] of black hole physics; (iii) the thin-wall approach to domain walls coupled to gravity.[55–58] More recently, *LL-branes* became significant also in the context of modern nonperturbative string theory, in particular, as the so-called H-branes describing quantum horizons (black hole and cosmological),[59] as Penrose limits of baryonic D-branes,[60] etc. (see also Refs. 61 and 62).

In the pioneering papers[55–58] *LL-branes* in the context of gravity and cosmology have been extensively studied from a phenomenological point of view, i.e. by introducing them without specifying the Lagrangian dynamics from which they may originate.[a] On the other hand, we have proposed in a series of recent papers[28–36] a new class of concise reparametrization invariant worldvolume Lagrangian actions (see also Sec. 4 below), providing a derivation from first principles of the *LL-brane* dynamics. The latter feature — the explicit worldvolume Lagrangian description of *LL-branes* is the principal distinction of our wormhole construction via (charged) *LL-branes* as sources of gravity and electromagnetism w.r.t. other non-Lagrangian "thin-shell" constructions of wormhole solutions (for the basics of the "thin-shell" cut-and-paste technique we refer to the book Ref. 19).

There are several characteristic features of *LL-branes* which drastically distinguish them from ordinary Nambu–Goto branes:

 (i) They describe intrinsically lightlike modes, whereas Nambu–Goto branes describe massive ones.

 (ii) The tension of the *LL-brane* arises as an *additional dynamical degree of freedom*, whereas Nambu–Goto brane tension is a given *ad hoc* constant. The latter characteristic feature significantly distinguishes our *LL-brane* models from the previously proposed *tensionless p-branes* (for a review of the latter, see Ref. 64) which rather resemble a *p*-dimensional continuous distribution of massless point-particles.

(iii) Consistency of *LL-brane* dynamics in a spherically or axially symmetric gravitational background of codimension one requires the presence of a horizon which is automatically occupied by the *LL-brane* ("horizon straddling" according to the terminology of Ref. 56).

(iv) When the *LL-brane* moves as a *test* brane in spherically or axially symmetric gravitational backgrounds its dynamical tension exhibits exponential "inflation/deflation" time behavior[65] — an effect similar to the "mass inflation" effect around black hole horizons.[66,67]

The principal object of study in the present paper is the self-consistently coupled gravity/nonlinear-gauge-field system, containing the square root of the Maxwell term, with one *or more LL-brane(s)*. We significantly extend the results of our previous note[27] by constructing both more general wormhole solutions displaying a "charge-hiding" effect as well as completely new "two-throat" wormhole solution with genuinely QCD-like *confining* behavior.

The plan of the present paper is as follows. In Sec. 2 we briefly review the Lagrangian formulation and the corresponding static spherically symmetric solutions of the coupled gravity/nonlinear-gauge-field system (2),[17] including the generation

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[a]In a more recent paper[63] brane actions in terms of their pertinent extrinsic geometry have been proposed which generically describe non-lightlike branes, whereas the lightlike branes are treated as a limiting case.

of vacuum constant-magnitude electric field as well as dynamical generation of cosmological constant.

In Sec. 3 we extend the results of Ref. 27 obtaining new solutions of compactified Levi-Civita–Bertotti–Robinson-type in the gravity/nonlinear-gauge-field system (2) depending on the magnitude of the bare cosmological constant versus the dynamically generated one.

In Sec. 4 we briefly review the worldvolume Lagrangian formulation and the basic properties of *LL-brane* dynamics, particularly stressing on the "horizon straddling" property (cf. Eqs. (42) below).

In Sec. 5 we describe the Lagrangian formulation of the self-consistently coupled bulk gravity/nonlinear-gauge-field system (2) to one or more *LL-brane* sources (cf. Eq. (43)) and outline a general procedure to derive wormhole solutions.

In Sec. 6 we construct "one-throat" wormhole solutions to the coupled gravity/ nonlinear-gauge-field/*LL-brane* system (43) with the charged *LL-brane* occupying the wormhole "throat," which connects a noncompact "universe" with Schwarzschild–(anti)-de Sitter geometry (where the cosmological constant is partially or entirely *dynamically* generated) to a compactified ("tube-like") "universe" of Levi-Civita–Bertotti–Robinson-type. These wormholes exhibit the novel property of *hiding* electric charge from external observer in the noncompact "universe," i.e. the whole electric flux produced by the charged *LL-brane* at the wormhole "throat" is expelled into the "tube-like" "universe."

In Sec. 7 we construct more general "two-throat" wormhole solution to the coupled gravity/nonlinear-gauge-field/*LL-brane* system (43) with two separate charged *LL-branes* with equal in magnitude but opposite charges occupying the wormhole "throats" and connecting pairwise three different "universes." The "left-most" and the "right-most" "universes" are two identical copies of the exterior region of the electrically neutral Schwarzschild–de Sitter black hole beyond the Schwarzschild horizon. The "middle" "universe" is of Levi-Civita–Bertotti–Robinson "tube-like" form with geometry $dS_2 \times S^2$ (dS_2 being the two-dimensional de Sitter space). It comprises the finite-extent intermediate region of dS_2 between its two horizons. Both oppositely charged *LL-branes* occupying the two "throats" are producing an electric flux which turns out to be *confined* entirely within the middle finite-extent "tube-like" "universe," i.e. no flux from the charged *LL-branes* is flowing into the noncompact outer "universes."

2. Gravity/Nonlinear-Gauge-Field System. Spherically Symmetric Solutions

We will consider the simplest coupling to gravity of the nonlinear gauge field system with a square root of the Maxwell term (1) known to produce QCD-like confinement in flat space–time.[3–7] The relevant action is given by (we use units with Newton constant $G_N = 1$):

$$S = \int d^4 x \sqrt{-G} \left[\frac{R(G) - 2\Lambda}{16\pi} + L(F^2) \right],$$

$$L(F^2) = -\frac{1}{4}F^2 - \frac{f}{2}\sqrt{\varepsilon F^2}\,,$$

$$F^2 \equiv F_{\kappa\lambda}F_{\mu\nu}G^{\kappa\mu}G^{\lambda\nu}\,, \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu\,. \tag{2}$$

Here $R(G)$ is the scalar curvature of the space–time metric $G_{\mu\nu}$ and $G \equiv \det\|G_{\mu\nu}\|$; the sign factor $\varepsilon = \pm 1$ in the square-root term in (2) corresponds to "magnetic" or "electric" dominance; f is a positive coupling constant.

It is important to stress that we *do not* need to introduce *any* bare cosmological constant Λ in (2) since the "square-root" Maxwell term dynamically generates a *nonzero effective cosmological constant* $\Lambda_{\text{eff}} = 2\pi f^2$.[27] The role of the bare Λ is just shifting the effective Λ_{eff} (see Eqs. (9) below).

Remark. One could start with the non-Abelian version of the gauge field action in (2). Since we will be interested in static spherically symmetric solutions, the non-Abelian gauge theory effectively reduces to an Abelian one as pointed out in Ref. 3.

The corresponding equations of motion read:

$$R_{\mu\nu} - \frac{1}{2}G_{\mu\nu}R + \Lambda G_{\mu\nu} = 8\pi T_{\mu\nu}^{(F)}\,, \tag{3}$$

$$T_{\mu\nu}^{(F)} = \left(1 + \varepsilon\frac{f}{\sqrt{\varepsilon F^2}}\right)F_{\mu\kappa}F_{\nu\lambda}G^{\kappa\lambda} - \frac{1}{4}\left(F^2 + 2f\sqrt{\varepsilon F^2}\right)G_{\mu\nu}\,, \tag{4}$$

and

$$\partial_\nu\left(\sqrt{-G}\left(1 + \varepsilon\frac{f}{\sqrt{\varepsilon F^2}}\right)F_{\kappa\lambda}G^{\mu\kappa}G^{\nu\lambda}\right) = 0\,. \tag{5}$$

Here we will first consider the case of "electric dominance," i.e. $\varepsilon = -1$ in (2). In our preceding paper[17] we have shown that the gravity–gauge-field system (2) with *zero* bare cosmological constant possesses static spherically symmetric solutions with a radial electric field containing *both* Coulomb and *constant* "vacuum" pieces:

$$F_{0r} = \frac{\varepsilon_F f}{\sqrt{2}} + \frac{Q}{\sqrt{4\pi}\,r^2}\,, \quad \varepsilon_F \equiv \text{sign}(F_{0r}) = \text{sign}(Q)\,, \tag{6}$$

and the space–time metric:

$$ds^2 = -A(r)dt^2 + \frac{dr^2}{A(r)} + r^2(d\theta^2 + \sin^2\theta\,d\varphi^2)\,, \tag{7}$$

$$A(r) = 1 - \sqrt{8\pi}|Q|f - \frac{2m}{r} + \frac{Q^2}{r^2} - \frac{2\pi f^2}{3}r^2\,, \tag{8}$$

is Reissner–Nordström–de Sitter-type with *dynamically generated* effective cosmological constant $2\pi f^2$. In the presence of the bare cosmological constant term in (2) the only effect is shifting of the effective cosmological constant, namely:

$$A(r) = 1 - \sqrt{8\pi}|Q|f - \frac{2m}{r} + \frac{Q^2}{r^2} - \frac{\Lambda_{\text{eff}}}{3}r^2\,, \quad \Lambda_{\text{eff}} = 2\pi f^2 + \Lambda\,. \tag{9}$$

The expression for Λ_{eff} (9) tells us that:

- Solution (6)–(7) with (9) is Reissner–Nordström–de Sitter-type with additional constant vacuum radial electric field even for *negative* bare cosmological constant $\Lambda < 0$ provided $|\Lambda| < 2\pi f^2$, i.e. $\Lambda_{\text{eff}} > 0$ in (9);
- Solution (6)–(7) with (9) becomes Reissner–Nordström-type with additional constant vacuum radial electric field in spite of the presence of *negative* bare cosmological constant $\Lambda < 0$ with $|\Lambda| = 2\pi f^2$, i.e. $\Lambda_{\text{eff}} = 0$ in (9);
- Solution (6)–(7) with (9) is Reissner–Nordström–*anti*-de Sitter-type with constant vacuum radial electric field for sufficiently large *negative* bare cosmological constant $\Lambda < 0$ with $|\Lambda| > 2\pi f^2$, i.e. $\Lambda_{\text{eff}} < 0$ in (9).

Notice that the "leading" constant term in the Reissner–Nordström–(anti-) de Sitter-type metric coefficient (9) is different from 1 when $Q \neq 0$. This effect resembles the effect on gravity produced by a spherically symmetric "hedgehog" configuration of a nonlinear sigma-model scalar field with $SO(3)$ symmetry (see Refs. 68 and 69).

The electrically neutral case $Q = 0$ will play an important role in what follows:

$$A(r) = 1 - \frac{2m}{r} - \frac{\Lambda_{\text{eff}}}{3}r^2, \quad \Lambda_{\text{eff}} = 2\pi f^2 + \Lambda, \quad F_{0r} = \frac{\varepsilon_F f}{\sqrt{2}}. \tag{10}$$

- Solution (10) is Schwarzschild–de Sitter black hole carrying a constant vacuum radial electric field for all $\Lambda > -2\pi f^2$, even for negative Λ provided $|\Lambda| < 2\pi f^2$, i.e. $\Lambda_{\text{eff}} > 0$ in (10).
- Solution (10) for negative Λ with $|\Lambda| = 2\pi f^2$ becomes asymptotically flat ordinary Schwarzschild carrying a constant vacuum radial electric field in spite of the presence of *negative* bare cosmological constant, i.e. $\Lambda_{\text{eff}} = 0$ in (10).
- Solution (10) is Schwarzschild–*anti*-de Sitter carrying a constant vacuum radial electric field for all $\Lambda < 0$ with $|\Lambda| > 2\pi f^2$, i.e. $\Lambda_{\text{eff}} < 0$ in (10).

3. Generalized Levi-Civita–Bertotti–Robinson Space–Times

Here we will look for static solutions of Levi-Civita–Bertotti–Robinson-type[37–39] of the system (3)–(5), namely, with space–time geometry of the form $\mathcal{M}_2 \times S^2$, where $\mathcal{M}_2$ is some two-dimensional manifold:

$$ds^2 = -A(\eta)dt^2 + \frac{d\eta^2}{A(\eta)} + r_0^2(d\theta^2 + \sin^2\theta\, d\varphi^2), \tag{11}$$

$$-\infty < \eta < \infty, \quad r_0 = \text{const},$$

and being:

- either purely electric type, where the sign factor $\varepsilon = -1$ in the gauge field Lagrangian $L(F^2)$ (2):

$$F_{\mu\nu} = 0 \quad \text{for } \mu, \nu \neq 0, \eta, \qquad F_{0\eta} = F_{0\eta}(\eta); \tag{12}$$

- or purely magnetic type, where $\varepsilon = +1$ in (2):

$$F_{\mu\nu} = 0 \quad \text{for } \mu,\, \nu \neq i,\, j \equiv \theta,\, \varphi\,, \qquad \partial_0 F_{ij} = \partial_\varphi F_{ij} = 0\,. \tag{13}$$

In the purely electric case (12) the gauge field equations of motion become:

$$\partial_\eta \left(F_{0\eta} - \frac{\varepsilon_F f}{\sqrt{2}} \right) = 0\,, \quad \varepsilon_F \equiv \mathrm{sign}(F_{0\eta})\,, \tag{14}$$

yielding a globally constant vacuum electric field:

$$F_{0\eta} = c_F = \text{arbitrary const}\,. \tag{15}$$

The (mixed) components of energy–momentum tensor (4) read:

$$T^{(F)0}{}_0 = T^{(F)\eta}{}_\eta = -\frac{1}{2} F_{0\eta}^2\,, \quad T^{(F)}_{ij} = g_{ij} \left(\frac{1}{2} F_{0\eta}^2 - \frac{f}{\sqrt{2}} |F_{0\eta}| \right)\,. \tag{16}$$

Taking into account (16), the Einstein equations (3) for (ij), where $R_{ij} = \frac{1}{r_0^2} g_{ij}$ because of the S^2 factor in (11), yield:

$$\frac{1}{r_0^2} = 4\pi c_F^2 + \Lambda\,. \tag{17}$$

The (00) Einstein equation (3) using the expression $R_0^0 = -\frac{1}{2}\partial_\eta^2 A$ (valid for metrics of the type (11), cf. Refs. 70 and 71) becomes:

$$\partial_\eta^2 A = 8\pi h(|c_F|)\,, \quad h(|c_F|) \equiv c_F^2 - \sqrt{2} f |c_F| - \frac{\Lambda}{4\pi}\,. \tag{18}$$

Thus, we arrive at the following three distinct types of Levi-Civita–Bertotti–Robinson solutions for gravity coupled to the nonlinear gauge field system (2):

(i) $\mathrm{AdS}_2 \times S^2$ with constant vacuum electric field $|F_{0\eta}| = |c_F|$, where AdS_2 is two-dimensional anti-de Sitter space with (using the definition of $h(|c_F|)$ in (18)):

$$A(\eta) = 4\pi h(|c_F|) \eta^2\,, \quad h(|c_F|) > 0 \tag{19}$$

in the metric (11), η being the Poincaré patch spacelike coordinate, provided:

$$|c_F| > \frac{f}{\sqrt{2}} \left(1 + \sqrt{1 + \frac{\Lambda}{2\pi f^2}} \right) \quad \text{for} \quad \Lambda \geq -2\pi f^2\,, \tag{20}$$

$$|c_F| > \sqrt{\frac{1}{4\pi} |\Lambda|} \qquad \qquad \text{for} \quad \Lambda < 0,\, |\Lambda| > 2\pi f^2\,. \tag{21}$$

(ii) $\mathrm{Rind}_2 \times S^2$ with constant vacuum electric field $|F_{0\eta}| = |c_F|$, where Rind_2 is the flat two-dimensional Rindler space with:

$$A(\eta) = \eta \text{ for } 0 < \eta < \infty \quad \text{or} \quad A(\eta) = -\eta \text{ for } -\infty < \eta < 0 \tag{22}$$

in the metric (11), provided:

$$|c_F| = \frac{f}{\sqrt{2}} \left(1 + \sqrt{1 + \frac{\Lambda}{2\pi f^2}} \right) \quad \text{for} \quad \Lambda \geq -2\pi f^2\,. \tag{23}$$

(iii) $dS_2 \times S^2$ with weak constant vacuum electric field $|F_{0\eta}| = |c_F|$, where dS_2 is two-dimensional de Sitter space with:

$$A(\eta) = 1 - K(|c_F|)\eta^2 \,,$$

$$K(|c_F|) \equiv -4\pi h(|c_F|) \equiv 4\pi \left(\sqrt{2} f |c_F| - c_F^2 + \frac{\Lambda}{4\pi} \right) > 0 \tag{24}$$

in the metric (11), provided:

$$|c_F| < \frac{f}{\sqrt{2}} \left(1 + \sqrt{1 + \frac{\Lambda}{2\pi f^2}} \right) \quad \text{for} \quad \Lambda > -2\pi f^2 \,. \tag{25}$$

When $\Lambda = 0$, for the special value $|c_F| = \frac{f}{\sqrt{2}}$ we recover the Nariai solution[72,73] with $A(\eta) = 1 - 2\pi f^2 \eta^2$ and equality (up to signs) among energy density, radial and transverse pressures: $\rho = -p_r = -p_\perp = \frac{f^2}{4}$ (using standard definitions $T^{(F)\mu}_{\quad\nu} = \mathrm{diag}(-\rho, p_r, p_\perp, p_\perp)$).

In all three cases above the size of the S^2 factor is given by (17). Solutions (22) and (24) with $\Lambda = 0$ are specifically due to the presence of the non-Maxwell square-root term (with $\varepsilon = -1$) in the gauge field Lagrangian (2).

In the purely magnetic case (13) the gauge field equations of motion (5):

$$\partial_\nu \left[\sin\theta \left(1 + \frac{f}{\sqrt{F^2}} \right) F^{\mu\nu} \right] = 0 \tag{26}$$

yield magnetic monopole solution:

$$F_{ij} = B r_0^2 \sin\theta \, \varepsilon_{ij} \,, \quad B = \text{const} \,, \tag{27}$$

irrespective of the presence of the "square-root" Maxwell term. The latter, however, does contribute to the energy–momentum tensor:

$$T^{(F)0}_{\quad 0} = T^{(F)\eta}_{\quad \eta} = -\frac{1}{2} B^2 - f|B| \,, \quad T^{(F)}_{ij} = \frac{1}{2} g_{ij} B^2 \,. \tag{28}$$

Taking into account (28), the Einstein equations (3) for (ij) yield (cf. (17)):

$$\frac{1}{r_0^2} = 4\pi \left(B^2 + \sqrt{2} f|B| \right) + \Lambda \,, \tag{29}$$

which determines the size of the S^2 factor, whereas the mixed-component (00) Einstein equation (3) gives:

$$\partial_\eta^2 A = 8\pi B^2 - 2\Lambda \,. \tag{30}$$

Thus, in the purely magnetic case we recover the three types of Levi-Civita–Bertotti–Robinson solutions with constant-magnitude magnetic field:

(a) $\mathrm{AdS}_2 \times S^2$ space–time with magnetic monopole (27) for $\Lambda < 4\pi B^2$ with $A(\eta) = 4\pi \left(B^2 - \frac{\Lambda}{4\pi} \right) \eta^2$ in the metric (11);

(b) $\text{Rind}_2 \times S^2$ space–time with magnetic monopole (27) for $\Lambda = 4\pi B^2$ with $A(\eta) = \eta$, $\eta > 0$ in the metric (11);

(c) $dS_2 \times S^2$ space–time with magnetic monopole (27) for $\Lambda > 4\pi B^2$ with $A(\eta) = 1 - 4\pi\left(\frac{\Lambda}{4\pi} - B^2\right)\eta^2$ in the metric (11).

Here the only feature is the dependence of the size of the S^2-factor on the "square-root" Maxwell coupling constant f (29).

Generalized Levi-Civita–Bertotti–Robinson solutions of the above type have already appeared in different contexts in Refs. 44–48 and 74 (extension to higher space–time dimensions). The main distinction in the present case is that the Levi-Civita–Bertotti–Robinson solutions are now generated due to the presence of the "square-root" Maxwell term in (2) which also produces a nonzero effective cosmological constant.

In Ref. 75 a different kind on nonlinear gauge field Lagrangian $L(F^2)$ coupled to gravity has been considered which generates *locally* (in the vicinity of the center of the geometry) an effective cosmological constant. However, the latter $L(F^2)$ is an *analytic* function of F^2 reducing to the ordinary Maxwell term for small F^2 unlike the present nonlinear Lagrangian $L(F^2)$ in (2) containing the square-root term $\sqrt{-F^2}$. This latter feature of (2) produces a *globally* defined dynamically generated cosmological constant $2\pi f^2$.

4. Lagrangian Formulation of Lightlike Branes. Horizon "Straddling"

In what follows we will consider gravity/gauge-field system self-consistently interacting with a lightlike p-brane ("*LL-brane*" for short) of codimension one ($D = (p + 1) + 1$); in the present section will keep arbitrary the number of space–time dimensions. In a series of previous papers[28–36] we have proposed manifestly reparametrization invariant worldvolume Lagrangian formulation in several dynamically equivalent forms of *LL-branes* coupled to bulk gravity $G_{\mu\nu}$ and bulk gauge fields, in particular, electromagnetic field A_μ. Here we will use our Polyakov-type formulation given by the worldvolume action:[35,36]

$$S_{\text{LL}}[q] = -\frac{1}{2} \int d^{p+1}\sigma \, T b_0^{\frac{p-1}{2}} \sqrt{-\gamma}\left[\gamma^{ab}\bar{g}_{ab} - b_0(p - 1)\right], \tag{31}$$

$$\bar{g}_{ab} \equiv \partial_a X^\mu G_{\mu\nu}\partial_b X^\nu - \frac{1}{T^2}(\partial_a u + q\mathcal{A}_a)(\partial_b u + q\mathcal{A}_b), \quad \mathcal{A}_a \equiv \partial_a X^\mu A_\mu. \tag{32}$$

Here and below the following notations are used:

- γ_{ab} is the *intrinsic* Riemannian metric on the worldvolume with $\gamma = \det\|\gamma_{ab}\|$; g_{ab} is the *induced* metric on the worldvolume:

$$g_{ab} \equiv \partial_a X^\mu G_{\mu\nu}(X)\partial_b X^\nu, \tag{33}$$

which becomes *singular* on-shell (manifestation of the lightlike nature, cf. Eq. (37) below); b_0 is a positive constant measuring the worldvolume "cosmological constant."

- $X^\mu(\sigma)$ are the p-brane embedding coordinates in the bulk D-dimensional space–time with Riemannian metric $G_{\mu\nu}(x)$ ($\mu,\nu = 0, 1, \ldots, D-1$); $(\sigma) \equiv (\sigma^0 \equiv \tau, \sigma^i)$ with $i = 1, \ldots, p$; $\partial_a \equiv \frac{\partial}{\partial\sigma^a}$.
- u is auxiliary worldvolume scalar field defining the lightlike direction of the induced metric (see Eq. (37) below) and it is a nonpropagating degree of freedom.[36]
- T is *dynamical (variable)* brane tension (also a nonpropagating degree of freedom).
- Coupling parameter q is the surface charge density of the *LL-brane*.

The corresponding equations of motion w.r.t. X^μ, u, γ_{ab} and T read accordingly (using short-hand notation (32)):

$$\partial_a(T\sqrt{|\bar{g}|}\bar{g}^{ab}\partial_b X^\mu) + T\sqrt{|\bar{g}|}\bar{g}^{ab}\partial_a X^\lambda \partial_b X^\nu \Gamma^\mu_{\lambda\nu}$$

$$+ \frac{q}{T}\sqrt{|\bar{g}|}\bar{g}^{ab}\partial_a X^\nu(\partial_b u + q\mathcal{A}_b)\mathcal{F}_{\lambda\nu}G^{\mu\lambda} = 0\,, \tag{34}$$

$$\partial_a\left(\frac{1}{T}\sqrt{|\bar{g}|}\bar{g}^{ab}(\partial_b u + q\mathcal{A}_b)\right) = 0\,, \quad \gamma_{ab} = \frac{1}{b_0}\bar{g}_{ab}\,, \tag{35}$$

$$T^2 + \bar{g}^{ab}(\partial_a u + q\mathcal{A}_a)(\partial_b u + q\mathcal{A}_b) = 0\,. \tag{36}$$

Here $\bar{g} = \det\|\bar{g}_{ab}\|$ and $\Gamma^\mu_{\lambda\nu}$ denotes the Christoffel connection for the bulk metric $G_{\mu\nu}$.

The on-shell singularity of the induced metric g_{ab} (33), i.e. the lightlike property, directly follows from Eq. (36) and the definition of $\bar{g}_{ab}$ (32):

$$g_{ab}\big(\bar{g}^{bc}(\partial_c u + q\mathcal{A}_c)\big) = 0\,. \tag{37}$$

Explicit worldvolume reparametrization invariance of the *LL-brane* action (31) allows to introduce the standard synchronous gauge-fixing conditions for the intrinsic worldvolume metric $\gamma_{00} = -1$, $\gamma_{0i} = 0$ ($i = 1, \ldots, p$), which reduces Eqs. (35) and (36) to the following relations:

$$\frac{(\partial_0 u + q\mathcal{A}_0)^2}{T^2} = b_0 + g_{00}\,, \quad \partial_i u + q\mathcal{A}_i = (\partial_0 u + q\mathcal{A}_0)g_{0i}(b_0 + g_{00})^{-1}\,,$$

$$g_{00} = g^{ij}g_{0i}g_{0j}\,, \quad \partial_0\big(\sqrt{g^{(p)}}\big) + \partial_i\big(\sqrt{g^{(p)}}g^{ij}g_{0j}\big) = 0\,, \quad g^{(p)} \equiv \det\|g_{ij}\|\,, \tag{38}$$

(recall that g_{00}, g_{0i}, g_{ij} are the components of the induced metric (33); g^{ij} is the inverse matrix of g_{ij}).

In our previous papers[28–36] we have studied in some detail the consistency of *LL-brane* dynamics in static "spherically-symmetric"-type backgrounds, whose generic form reads (in what follows we will use Eddington–Finkelstein coordinates[76,77] where $dt = dv - \frac{d\eta}{A(\eta)}$, $F_{0\eta} = F_{v\eta}$):

$$ds^2 = -A(\eta)dv^2 + 2\,dv\,d\eta + C(\eta)h_{ij}(\theta)d\theta^i\,d\theta^j \quad F_{v\eta} = F_{v\eta}(\eta)\,, \tag{39}$$

all remaining components of $F_{\mu\nu}$ being zero. For the *LL-brane* we use the standard embedding ansatz:

$$X^0 \equiv v = \tau \,, \quad X^1 \equiv \eta = \eta(\tau) \,, \quad X^i \equiv \theta^i = \sigma^i \quad (i = 1, \ldots, p) \,. \tag{40}$$

For the class of backgrounds (39) with the embedding (40) (where the induced metric components $g_{0i} = 0$) Eqs. (38) reduce to:

$$g_{00} = 0 \,, \quad \partial_0 C(\eta(\tau)) \equiv \dot{\eta}\partial_\eta C|_{\eta=\eta(\tau)} = 0 \,, \quad \frac{(\partial_0 u + q\mathcal{A}_0)^2}{T^2} = b_0 \,, \quad \partial_i u = 0 \tag{41}$$

($\dot{\eta} \equiv \partial_0 \eta \equiv \partial_\tau \eta(\tau)$). Thus, in the generic case of nontrivial dependence of $C(\eta)$ on the "radial-like" coordinate η, the first two relations in (41) yield:

$$\dot{\eta} = \frac{1}{2}A(\eta(\tau)) \,, \quad \dot{\eta} = 0 \to \eta(\tau) = \eta_0 = \text{const} \,, \quad A(\eta_0) = 0 \,. \tag{42}$$

In other words, consistency of *LL-brane* dynamics requires the corresponding background (39) to possess a horizon at some $\eta = \eta_0$, which is *automatically occupied* by the *LL-brane*.

The latter property is called "horizon straddling" according to the terminology of Ref. 56. Similar "horizon straddling" has been found also for *LL-branes* moving in rotating axially symmetric (Kerr or Kerr–Newman) and rotating cylindrically symmetric black hole backgrounds.[33,34]

5. Bulk Gravity/Nonlinear-Gauge-Field System Coupled to Lightlike Brane Sources

We consider now bulk Einstein/nonlinear gauge field system (2) self-consistently coupled to $N \geq 1$ distantly separated charged codimension-one *lightlike* branes (in the present case $D = 4$, $p = 2$). The pertinent Lagrangian action reads:

$$S = \int d^4x \sqrt{-G}\left[\frac{R(G) - 2\Lambda}{16\pi} + L(F^2)\right] + \sum_{k=1}^{N} S_{\text{LL}}\big[q^{(k)}\big] \,,$$
$$L(F^2) = -\frac{1}{4}F^2 - \frac{f}{2}\sqrt{-F^2} \,, \tag{43}$$

where $S_{\text{LL}}[q^{(k)}]$ indicates the worldvolume action of the kth *LL-brane* of the form (31). Henceforth we will consider the case of "electric dominance" for the "square-root" Maxwell term.

The corresponding equations of motion are as follows:

$$R_{\mu\nu} - \frac{1}{2}G_{\mu\nu}R + \Lambda G_{\mu\nu} = 8\pi\left[T_{\mu\nu}^{(F)} + \sum_{k=1}^{N} T_{\mu\nu}^{(\text{brane-}k)}\right] \,, \tag{44}$$

$$\partial_\nu\left[\sqrt{-G}\left(1 - \frac{f}{\sqrt{-F^2}}\right)F_{\kappa\lambda}G^{\mu\kappa}G^{\nu\lambda}\right] + \sum_{k=1}^{N} j_{(\text{brane-}k)}^{\mu} = 0 \,. \tag{45}$$

Here $T_{\mu\nu}^{(F)}$ is the same as in (4), whereas the energy–momentum tensor and the charge current density of kth *LL-brane* are straightforwardly derived from the pertinent *LL-brane* action (31):

$$
\begin{aligned}
T_{(\text{brane-}k)}^{\mu\nu} = & -\int d^3\sigma \, \frac{\delta^{(4)}(x - X_{(k)}(\sigma))}{\sqrt{-G}} \, T^{(k)} \\
& \times \sqrt{|\bar{g}_{(k)}|} \bar{g}_{(k)}^{ab} \partial_a X_{(k)}^\mu \partial_b X_{(k)}^\nu \,,
\end{aligned}
\tag{46}
$$

$$
\begin{aligned}
j_{(\text{brane-}k)}^\mu = & -q^{(k)} \int d^3\sigma \, \delta^{(4)}(x - X_{(k)}(\sigma)) \\
& \times \sqrt{|\bar{g}_{(k)}|} \bar{g}_{(k)}^{ab} \partial_a X_{(k)}^\mu \frac{\partial_b u^{(k)} + q^{(k)} \mathcal{A}_b^{(k)}}{T^{(k)}} \,,
\end{aligned}
\tag{47}
$$

where for each kth *LL-brane*:

$$
\begin{aligned}
& \bar{g}_{ab}^{(k)} \equiv g_{ab}^{(k)} - \frac{1}{T_{(k)}^2} \Big(\partial_a u^{(k)} + q^{(k)} \mathcal{A}_a^{(k)} \Big) \Big(\partial_b u^{(k)} + q^{(k)} \mathcal{A}_b^{(k)} \Big) \,, \\
& g_{ab}^{(k)} = \partial_a X_{(k)}^\mu G_{\mu\nu} \partial_b X_{(k)}^\nu \,, \quad \mathcal{A}_a^{(k)} \equiv \partial_a X_{(k)}^\mu A_\mu \,.
\end{aligned}
\tag{48}
$$

The *LL-brane* equations of motion have already been written down in (34)–(36) above.

Constructing wormhole solutions of static "spherically-symmetric"-type (39) for the coupled gravity-gauge-field-*LL-brane* system (43) proceeds through the following steps:

(i) Choose "vacuum" static "spherically-symmetric"-type solutions (39) of (44)–(45), i.e. without the delta-function terms due to the *LL-branes*, in each space–time region (separate "universe") given by $\left(-\infty < \eta < \eta_0^{(1)}\right)$, $\left(\eta_0^{(1)} < \eta < \eta_0^{(2)}\right), \ldots, \left(\eta_0^{(N)} < \eta < \infty\right)$ with common horizon(s) at $\eta = \eta_0^{(k)}$ $(k = 1, \ldots, N)$.

(ii) Each kth *LL-brane* automatically locates itself on the horizon at $\eta = \eta_0^{(k)}$ according to "horizon straddling" property (42) of *LL-brane* dynamics. It thus will play the role of a wormhole "throat" between two neighboring "universes."

(iii) Match the discontinuities of the derivatives of the metric and the gauge field strength (39) across each horizon at $\eta = \eta_0^{(k)}$ using the explicit expressions for the *LL-brane* stress–energy tensor and charge current density (46)–(47).

Taking into account (39)–(42), we obtain from (46) the following expression for the energy–momentum tensor of each kth *LL-brane* (here we suppress the index (k)):

$$
T_{(\text{brane})}^{\mu\nu} = S^{\mu\nu} \delta(\eta - \eta_0)
\tag{49}
$$

with surface energy–momentum tensor:

$$S^{\mu\nu} \equiv \frac{T}{b_0^{1/2}} \left(\partial_\tau X^\mu \partial_\tau X^\nu - b_0 G^{ij} \partial_i X^\mu \partial_j X^\nu \right)_{v=\tau, \eta=\eta_0, \theta^i = \sigma^i}, \tag{50}$$

where $G_{ij} = C(\eta) h_{ij}(\theta)$ (cf. (39), here $i, j = \theta, \varphi$). For the nonzero components of (50) (with lower indices) and its trace we find:

$$S_{\eta\eta} = \frac{T}{b_0^{1/2}} \quad S_{ij} = -T b_0^{1/2} G_{ij}, \quad S_\lambda^\lambda = -2 T b_0^{1/2}. \tag{51}$$

For the *LL-brane* charge current densities we get accordingly:

$$j^\mu_{(\text{brane-}k)} = \delta_0^\mu q^{(k)} \sqrt{\det \|G_{ij}\|} \, \delta(\eta - \eta_0^{(k)}). \tag{52}$$

With the help of (49)–(52) and using again (39)–(42) the matching relations for the discontinuities at each horizon $\eta = \eta_0^{(k)}$ become (cf. Refs. 35 and 36):

(A) Matching relations from Einstein equations (44):

$$\left[\partial_\eta A \right]_{\eta_0^{(k)}} = -16\pi T^{(k)} \sqrt{b_0^{(k)}}, \quad \left[\partial_\eta \ln C \right]_{\eta_0^{(k)}} = -\frac{8\pi}{\sqrt{b_0^{(k)}}} T^{(k)} \tag{53}$$

using notation $\left[Y \right]_{\eta_0} \equiv Y|_{\eta \to \eta_0 + 0} - Y|_{\eta \to \eta_0 - 0}$ for any quantity Y.

(B) Matching relations from nonlinear gauge field equations (45):

$$\left[F_{v\eta} \right]_{\eta_0^{(k)}} = q^{(k)}. \tag{54}$$

(C) The only nontrivial contribution of second-order *LL-brane* equations of motion (34) in the case of *LL-brane* coordinate embedding (40) comes from the X^0-equation of motion which yields:

$$\partial_0 T^{(k)} + \frac{T^{(k)}}{2} \left(\langle \partial_\eta A \rangle_{\eta_0^{(k)}} + 2 b_0^{(k)} \langle \partial_\eta \ln C \rangle_{\eta_0^{(k)}} \right) - \sqrt{b_0^{(k)}} q \langle F_{v\eta} \rangle_{\eta_0^{(k)}} = 0 \tag{55}$$

with notation $\langle Y \rangle_{\eta_0} \equiv \frac{1}{2} \left(Y|_{\eta \to \eta_0 + 0} + Y|_{\eta \to \eta_0 - 0} \right)$. In what follows we will take time-independent dynamical *LL-brane* tension(s) ($\partial_0 T^{(k)} = 0$) because of matching static bulk space–time geometries. Let us also note that the appearance of mean values of the corresponding quantities with discontinuities across the horizons follows the resolution of the discontinuity problem given in Ref. 55 (see also Ref. 78).

The wormhole solutions presented in the next section share the following important properties:

(a) The *LL-branes* at the wormhole "throats" represent "exotic" matter with $T \leq 0$, i.e. negative or zero brane tension implying violation of null-energy conditions as predicted by general wormhole arguments[19] (although the latter could be remedied via quantum fluctuations).

(b) The wormhole space–times constructed via *LL-branes* at their "throats" are *not* traversable w.r.t. the "laboratory" time of a static observer in either of the different "universes" comprising the pertinent wormhole space–time manifold since the *LL-branes* sitting at the "throats" look as black hole horizons to the static observer. On the other hand, these wormholes *are traversable* w.r.t. the *proper time* of a traveling observer.

Proper-time traversability can be easily seen by considering dynamics of test particle of mass m_0 ("traveling observer") in a wormhole background, which is described by the reparametrization-invariant worldline action:

$$S_{\text{particle}} = \frac{1}{2} \int d\lambda \left[\frac{1}{e} \dot{x}^\mu \dot{x}^\nu G_{\mu\nu} - e m_0^2 \right]. \tag{56}$$

Using energy $\mathcal{E}$ and orbital momentum $\mathcal{J}$ conservation and introducing the *proper* worldline time s $\left(\frac{ds}{d\lambda} = e m_0 \right)$, the "mass-shell" constraint equation (the equation w.r.t. the "einbein" e) produced by the action (56)) yields:

$$\left(\frac{d\eta}{ds} \right)^2 + \mathcal{V}_{\text{eff}}(\eta) = \frac{\mathcal{E}^2}{m_0^2}, \quad \mathcal{V}_{\text{eff}}(\eta) \equiv A(\eta) \left(1 + \frac{\mathcal{J}^2}{m_0^2 C(\eta)} \right), \tag{57}$$

where the metric coefficients $A(\eta)$, $C(\eta)$ are those in (39). Irrespectively of the specific form of the "effective potential" in (57), a "radially" moving (with zero "impact" parameter $\mathcal{J} = 0$) traveling observer (and with sufficiently large energy $\mathcal{E}$) will always cross within finite amount of proper-time through any "throat" ($\eta = \eta_0^{(k)}$, where $A(\eta_0^{(k)}) = 0$) from one "universe" to another.

6. Charge-Hiding Wormholes

First we will construct "one-throat" wormhole solutions to (43) with the charged *LL-brane* occupying the wormhole "throat," which connects a noncompact "universe" with Reissner–Nordström–(anti)-de Sitter-type geometry (6)–(8) (where the cosmological constant is partially or entirely *dynamically* generated) to a compactified ("tube-like") "universe" of (generalized) Levi-Civita–Bertotti–Robinson-type (11)–(12). These wormholes possess the novel property of *hiding* electric charge from external observer in the noncompact "universe," i.e. the whole electric flux produced by the charged *LL-brane* at the wormhole "throat" is pushed into the "tube-like" "universe." As a result, the noncompact "universe" becomes electrically neutral with Schwarzschild–(anti-)de Sitter or purely Schwarzschild geometry.

We find several types of such wormhole solutions. The first one exists when the bare cosmological constant $\Lambda > -2\pi f^2$, in particular, when Λ is absent from the very beginning, the whole effective cosmological constant being dynamically generated (9). This wormhole solution is constructed as follows:

(A-1) "Left universe" of Levi-Civita–Bertotti–Robinson ("tube-like") type with geometry $\text{AdS}_2 \times S^2$ (19) for $\eta < 0$:

$$A(\eta) = 4\pi \left(c_F^2 - \sqrt{2}f|c_F| - \frac{\Lambda}{4\pi} \right)\eta^2 \,,$$

$$C(\eta) \equiv r_0^2 = \frac{1}{4\pi c_F^2 + \Lambda} = \text{const} \,, \tag{58}$$

$$|F_{v\eta}| = |c_F| > \frac{f}{\sqrt{2}} \left(1 + \sqrt{1 + \frac{\Lambda}{2\pi f^2}} \right) = \text{const} \,;$$

(A-2) Noncompact "right universe" for $\eta > 0$ comprising the exterior region of Reissner–Nordström–de Sitter-type black hole beyond the middle (Schwarzschild-type) horizon r_0 (cf. (6)–(9)):

$$A(\eta) \equiv A_{\text{RNdS}}(r_0 + \eta)$$

$$= 1 - \sqrt{8\pi}|Q|f - \frac{2m}{r_0 + \eta} + \frac{Q^2}{(r_0 + \eta)^2}$$

$$\quad - \frac{\Lambda + 2\pi f^2}{3}(r_0 + \eta)^2 \,, \tag{59}$$

$$C(\eta) = (r_0 + \eta)^2 \,,$$

$$F_{v\eta} = F_{0r} = \frac{\varepsilon_F f}{\sqrt{2}} + \frac{Q}{\sqrt{4\pi}\,(r_0 + \eta)^2} \,.$$

Here $A(0) = A_{\text{RNdS}}(r_0) = 0$ and $\partial_\eta A(0) = \partial_r A_{\text{RNdS}}(r_0) > 0$.

The next wormhole solution, which exists for large negative bare cosmological constant $\Lambda < 0$, $|\Lambda| > 2\pi f^2$, is built by:

(B-1) The same type of "left universe" of Levi-Civita–Bertotti–Robinson-type with geometry $\text{AdS}_2 \times S^2$ (19) for $\eta < 0$ as in (58):

$$A(\eta) = 4\pi \left(c_F^2 - \sqrt{2}f|c_F| + \frac{|\Lambda|}{4\pi} \right)\eta^2 \,,$$

$$C(\eta) \equiv r_0^2 = \frac{1}{4\pi c_F^2 - |\Lambda|} = \text{const} \,, \tag{60}$$

$$|F_{v\eta}| = |c_F| > \sqrt{\frac{1}{4\pi}|\Lambda|} \,.$$

(B-2) Noncompact "right universe" for $\eta > 0$ comprising the exterior region of Reissner–Nordström–*anti*-de Sitter-type black hole beyond the outer

(Schwarzschild-type) horizon r_0:

$$A(\eta) \equiv A_{\text{RN–AdS}}(r_0 + \eta)$$

$$= 1 - \sqrt{8\pi}|Q|f - \frac{2m}{r_0 + \eta} + \frac{Q^2}{(r_0 + \eta)^2}$$

$$+ \frac{|\Lambda| - 2\pi f^2}{3}(r_0 + \eta)^2 \,, \tag{61}$$

$$C(\eta) = (r_0 + \eta)^2 \,,$$

$$F_{v\eta} = F_{0r} = \frac{\varepsilon_F f}{\sqrt{2}} + \frac{Q}{\sqrt{4\pi}\,(r_0 + \eta)^2} \,.$$

Here again $A(0) = A_{\text{RN–AdS}}(r_0) = 0$ and $\partial_\eta A(0) = \partial_r A_{\text{RN–AdS}}(r_0) > 0$.

For the special negative value of the bare cosmological constant $\Lambda = -2\pi f^2$ we find a third wormhole solution consisting of:

(C-1) The same type of "left universe" of Levi-Civita–Bertotti–Robinson-type with geometry $\text{AdS}_2 \times S^2$ (19) for $\eta < 0$ as in (58):

$$A(\eta) = 4\pi \left(|c_F| - \frac{f}{\sqrt{2}} \right)^2 \eta^2 \,,$$

$$C(\eta) \equiv r_0^2 = \frac{1}{4\pi \left(c_F^2 - \frac{1}{2}f^2 \right)} = \text{const} \,, \tag{62}$$

$$|F_{v\eta}| = |c_F| > \frac{f}{\sqrt{2}} \,.$$

(C-2) Noncompact "right universe" for $\eta > 0$ comprising the exterior region of Reissner–Nordström-type black hole beyond the outer (Schwarzschild-type) horizon r_0:

$$A(\eta) \equiv A_{\text{RN}}(r_0 + \eta) = 1 - \sqrt{8\pi}|Q|f - \frac{2m}{r_0 + \eta} + \frac{Q^2}{(r_0 + \eta)^2} \,,$$

$$\tag{63}$$

$$C(\eta) = (r_0 + \eta)^2 \,, \quad F_{v\eta} = F_{0r} = \frac{\varepsilon_F f}{\sqrt{2}} + \frac{Q}{\sqrt{4\pi}\,(r_0 + \eta)^2} \,.$$

Here again $A(0) = A_{\text{RN}}(r_0) = 0$ and $\partial_\eta A(0) = \partial_r A_{\text{RN}}(r_0) > 0$.

Substituting (58)–(59), (60)–(61) and (62)–(63) in the set of matching relations (53)–(55) determines all parameters of the wormhole solutions (r_0, m, Q, b_0, T) in terms of q (the *LL-brane* charge) and f (coupling constant of the "square-root" Maxwell term in (43)):

$$Q = 0 \,, \quad |c_F| = |q| + \frac{f}{\sqrt{2}} \,,$$

$$\tag{64}$$

$$\text{sign}(q) = -\text{sign}(F_{v\eta}) \equiv -\text{sign}(c_F) \,,$$

$$\frac{1}{r_0^2} = 4\pi\left(|q| + \frac{f}{\sqrt{2}}\right)^2 + \Lambda\,,$$

$$m = \frac{r_0}{2}\left[1 - \frac{1}{3}(\Lambda + 2\pi f^2)r_0^2\right]\,,$$

(65)

$$b_0 = \frac{1}{4}\left(q^2 + \sqrt{2}f|q|\right)\left[\left(|q| + \frac{f}{\sqrt{2}}\right)^2 + \frac{1}{4\pi}\Lambda\right]^{-1}\,,$$

$$T^2 = \frac{1}{16\pi}\left(q^2 + \sqrt{2}f|q|\right)\,,$$

(66)

and the bare cosmological constant must be in the interval:

$$-4\pi\left(|q| + \frac{f}{\sqrt{2}}\right)^2 < \Lambda < 4\pi\left(q^2 - \frac{f^2}{2}\right)\,,$$

(67)

in particular, Λ could be zero.

The next wormhole solution has $\mathrm{Rind}_2 \times S^2$ as compactified "left" universe whenever $\Lambda > -2\pi f^2$. It is built by:

(D-1) "Left universe" for $\eta < 0$ of Levi-Civita–Bertotti–Robinson ("tube-like") type with geometry $\mathrm{Rind}_2 \times S^2$ (22):

$$A(\eta) = -\eta\,, \quad C(\eta) \equiv r_0^2 = \frac{1}{4\pi c_F^2 + \Lambda} = \mathrm{const}\,,$$

$$|F_{v\eta}| = |c_F| = \frac{f}{\sqrt{2}}\left(1 + \sqrt{1 + \frac{\Lambda}{2\pi f^2}}\right) = \mathrm{const}\,;$$

(68)

(D-2) Noncompact "right universe" for $\eta > 0$ comprising the exterior region of Reissner–Nordström–de Sitter-type black hole beyond the middle (Schwarzschild-type) horizon r_0 as in (59).

Again substituting (68) and (59) into the set of matching relations (53)–(55) determines all parameters of the wormhole solution (D-1)–(D-2) in complete analogy with (64)–(66):

$$Q = 0\,, \quad |c_F| = |q| + \frac{f}{\sqrt{2}}\,, \quad \mathrm{sign}(q) = -\mathrm{sign}(F_{v\eta}) \equiv -\mathrm{sign}(c_F)\,,$$

(69)

$$\Lambda = 4\pi\left(q^2 - \frac{f^2}{2}\right)\,, \quad \frac{1}{r_0^2} = 8\pi\left(q^2 + \frac{f}{\sqrt{2}}|q|\right)\,, \quad m = \frac{r_0}{2}\left[1 - \frac{4\pi q^2}{3}r_0^2\right]\,,$$

(70)

$$b_0 = \frac{1}{4}\left[1 + r_0 - 4\pi q^2 r_0^2\right]\,, \quad T^2 = \frac{b_0}{2\pi}\left(q^2 + \frac{f}{\sqrt{2}}|q|\right)\,.$$

(71)

The result $Q = 0$ in (64) and (69) has profound consequences. Namely, the absence of Coulomb field in spite of the presence of the charged *LL-brane* source leads us to the following important observations:

(A) The "right-universe" in the wormhole solutions (A-1)–(A-2) (Eqs. (58)–(59)) and (D-1)–(D-2) (Eqs. (68), (59)) becomes exterior region of electrically neutral Schwarzschild–de Sitter black hole beyond the internal (Schwarzschild-type) horizon carrying a vacuum constant radial electric field $|F_{v\eta}| = |F_{0r}| = \frac{f}{\sqrt{2}}$.

(B) The "right-universe" in the wormhole solution (B-1)–(B-2) (Eqs. (60)–(61)) becomes exterior region of electrically neutral Schwarzschild–*anti*-de Sitter black hole beyond the sole (Schwarzschild-type) horizon carrying a vacuum constant radial electric field $|F_{v\eta}| = |F_{0r}| = \frac{f}{\sqrt{2}}$.

(C) The "right-universe" in the wormhole solution (C-1)–(C-2) (Eqs. (62)–(63)) becomes exterior region of the ordinary electrically neutral Schwarzschild black hole beyond the horizon carrying a vacuum constant radial electric field $|F_{v\eta}| = |F_{0r}| = \frac{f}{\sqrt{2}}$.

(D) According to (64) and (69) the whole flux of the electric field $|F_{0\eta}|$ with $|F_{0\eta}| = |F_{v\eta}| = \frac{f}{\sqrt{2}} + |q|$ produced by the *LL-brane* charge q flows only into the compactified "left universe" of Levi-Civita–Bertotti–Robinson-type ($AdS_2 \times S^2$ (19) or $Rind_2 \times S^2$ (22)). Due to the absence of electric flux in the noncompact "right universe," an outside observer there will therefore detect the charged *LL-brane* as a neutral object.

A clearer explanation of above statements (A)–(D) can be given if we recall that the electric flux is defined in terms of the electric displacement field $\mathbf{D}$, which in the present case is significantly different from the electric field $\mathbf{E}$ due to the presence of the "square-root" Maxwell term in (43):

$$\mathbf{D} = \left(1 - \frac{f}{\sqrt{2}|\mathbf{E}|}\right)\mathbf{E} \,. \tag{72}$$

Indeed, in the absence of magnetic field the 0th component of the nonlinear gauge field equations (45) can be written in terms of $\mathbf{D}$ (72) as:

$$\boldsymbol{\partial} \cdot \left(\sqrt{-G}\mathbf{D}\right) - \sqrt{-G}J^0 = 0 \,, \tag{73}$$

where $J^\mu = \frac{1}{\sqrt{-G}}j^\mu$ is the charge vector current, so that:

$$\int_{\partial\Sigma} d\mathbf{S} \cdot \mathbf{D} = Q_{\text{total}} = \int_\Sigma dV \, J^0 \,. \tag{74}$$

Here the factors $\sqrt{-G}$ go into the definition of the corresponding volume forms (integration measures) on the three-dimensional region Σ and its boundary $\partial\Sigma$.

Thus, Eq. (74) tells us that the electric flux from the charged *LL-brane* flowing into the noncompact "right universes," where the constant radial vacuum electric field has magnitude $|\mathbf{E}| = \frac{f}{\sqrt{2}}$, is *zero* since $\mathbf{D} = 0$ there according to (72). On the other hand, inside the compactified Levi-Civita–Bertotti–Robinson-type "left universe":

$$\mathbf{D} = \frac{|q|}{\frac{f}{\sqrt{2}} + |q|}\mathbf{E} = -q\hat{\eta} \,, \tag{75}$$
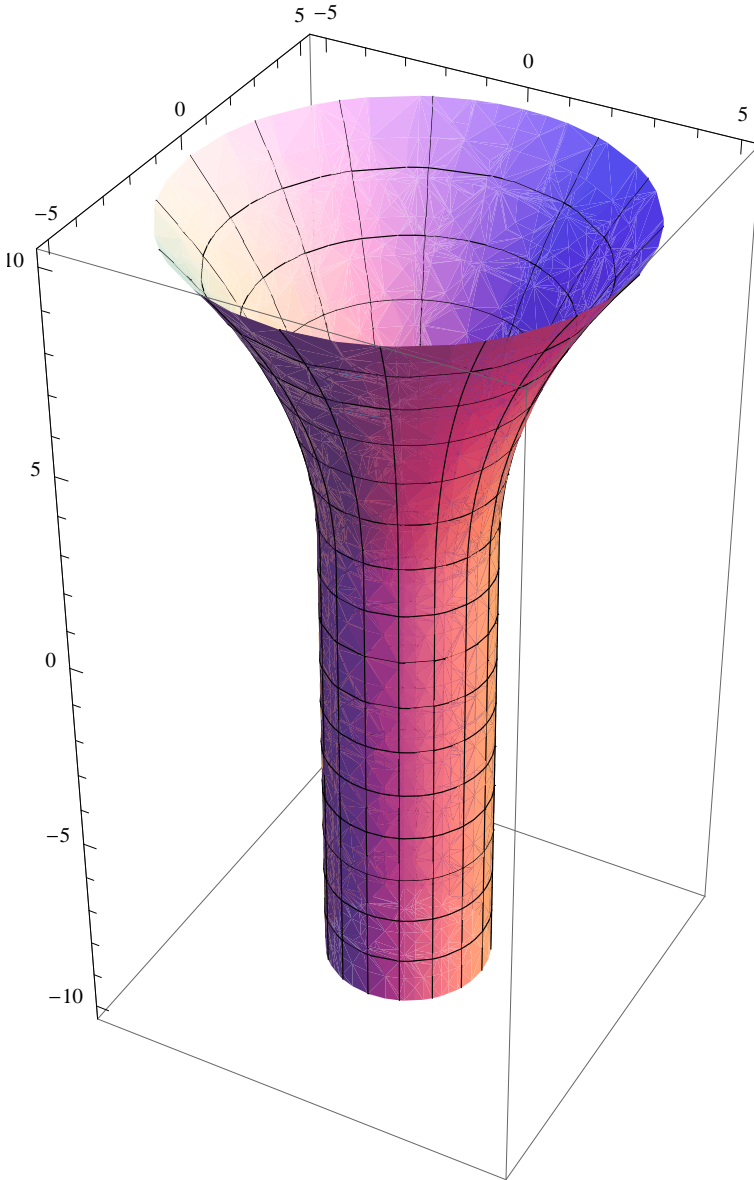
Fig. 1.   Shape of $t = \text{const}$ and $\theta = \frac{\pi}{2}$ slice of charge-"hiding" wormhole geometry. The whole electric flux is expelled into the lower cylindric tube.

where $\hat{\eta}$ denotes the unit vector along the "radial-like" η coordinate (here we have used relations (64) and (69)). Therefore, the whole electric flux from the charged *LL-brane* is expelled into the "tube-like" "left universe."

The geometry of the charge-"hiding" wormholes is visualized in Fig. 1.

7. Charge-Confining Wormhole

Apart from the above *charge-hiding effect* produced by "one-throat" wormhole connecting a noncompact "universe" to a compactified "tube-like" "universe" via *LL-brane* we find an even more interesting "two-throat" wormhole solution exhibiting *QCD-like charge confinement* effect. Namely, let us now consider a self-consistent coupling of the gravity/nonlinear-gauge-field system (2) with two separate *oppositely charged*, but otherwise identical *LL-branes* described by the action (43) and the resulting equations of motion (44)–(48) (here $N = 2$, $T^{(1)} = T^{(2)} \equiv T$, $b_0^{(1)} = b_0^{(2)} \equiv b_0$, $q^{(1)} \equiv q = -q^{(2)}$).

Using the general scheme outlined in Sec. 5 we construct a solution where the total "two-throat" wormhole space–time manifold is built as follows:

(E-1) "Left-most" noncompact "universe" comprising the exterior region of Reissner–Nordström–de Sitter-type black hole beyond the middle Schwarzschild-type horizon r_0 for the "radial-like" η-coordinate interval (see also Eqs. (79) and (82) below):

$$-\infty < \eta < -\eta_0 \equiv -\left[4\pi(\sqrt{2}f|c_F| - c_F^2) + \Lambda\right]^{-\frac{1}{2}}, \tag{76}$$

where (cf. (6)–(9)):

$$A(\eta) = A_{\mathrm{RNdS}}(r_0 - \eta_0 - \eta)$$

$$= 1 - \sqrt{8\pi}|Q|f - \frac{2m}{r_0 - \eta_0 - \eta} + \frac{Q^2}{(r_0 - \eta_0 - \eta)^2}$$

$$- \frac{\Lambda + 2\pi f^2}{3}(r_0 - \eta_0 - \eta)^2, \tag{77}$$

$$C(\eta) = (r_0 - \eta_0 - \eta)^2,$$

$$F_{v\eta}(\eta) = F_{0r}(\eta) = \frac{\varepsilon_F f}{\sqrt{2}} + \frac{Q}{\sqrt{4\pi}\,(r_0 - \eta_0 - \eta)^2}. \tag{78}$$

Here $A(-\eta_0) = A_{\mathrm{RNdS}}(r_0) = 0$ and $\partial_\eta A(-\eta_0) = -\partial_r A_{\mathrm{RNdS}}(r_0) < 0$.

(E-2) "Middle" compactified "tube-like" "universe" of Levi-Civita–Bertotti–Robinson-type with geometry $dS_2 \times S^2$ (24)–(25) comprising the finite extent (w.r.t. η-coordinate) region between the two horizons of dS_2 at $\eta = \pm\eta_0$:

$$-\eta_0 < \eta < \eta_0 \equiv \left[4\pi(\sqrt{2}f|c_F| - c_F^2) + \Lambda\right]^{-\frac{1}{2}}, \tag{79}$$

where (cf. Eqs. (24)–(25)):

$$A(\eta) = 1 - \left[4\pi(\sqrt{2}f|c_F| - c_F^2) + \Lambda\right]\eta^2, \quad A(\pm\eta_0) = 0, \tag{80}$$

$$C(\eta) = r_0^2 = \frac{1}{4\pi c_F^2 + \Lambda}, \quad |F_{v\eta}| = |c_F| < \frac{f}{\sqrt{2}}\left(1 + \sqrt{1 + \frac{\Lambda}{2\pi f^2}}\right), \tag{81}$$

with $\Lambda > -2\pi f^2$;

(E-3) "Right-most" noncompact "universe" comprising the exterior region of Reissner–Nordström–de Sitter-type black hole beyond the middle Schwarzschild-type horizon r_0 for the "radial-like" η-coordinate interval:

$$\eta_0 < \eta < \infty \quad (\eta_0 \text{ as in (79)}),\tag{82}$$

i.e. a mirror image $(-\eta \to \eta)$ of the "left-most" "universe" (77)–(78):

$$A(\eta) = A_{\mathrm{RNdS}}(r_0 + \eta - \eta_0)$$

$$= 1 - \sqrt{8\pi}|Q|f - \frac{2m}{r_0 + \eta - \eta_0} + \frac{Q^2}{(r_0 + \eta - \eta_0)^2}$$

$$- \frac{\Lambda + 2\pi f^2}{3}(r_0 + \eta - \eta_0)^2,\tag{83}$$

$$C(\eta) = (r_0 + \eta - \eta_0)^2,$$

$$F_{v\eta}(\eta) = F_{0r}(\eta) = \frac{\varepsilon_F f}{\sqrt{2}} + \frac{Q}{\sqrt{4\pi}(r_0 + \eta - \eta_0)^2}.\tag{84}$$

Here $A(\eta_0) = A_{\mathrm{RNdS}}(r_0) = 0$ and $\partial_\eta A(\eta_0) = \partial_r A_{\mathrm{RNdS}}(r_0) > 0$.

According to the "horizon straddling" property (42) of worldvolume *LL-brane* dynamics, each one of the two charged *LL-branes* (with equal worldvolume parameters (T, b_0) but with opposite charges $\pm q$, cf. (31)), automatically locates itself on one of the two common horizons between "left-most" (E-1) and middle (E-2) "universes" at $\eta = -\eta_0$ and between middle (E-2) and "right-most" (E-3) "universes" at $\eta = \eta_0$, respectively.

Now, as we did in the previous section, substituting (76)–(84) into the set of matching relations (53)–(55) determines all parameters of the wormhole solutions $(r_0, \eta_0, m, Q, b_0, T)$ in terms of $|q|$ (the magnitude of the *LL-brane* charges) and f (the coupling constant of the "square-root" Maxwell term in (43)):

$$Q = 0, \quad |c_F| = |q| + \frac{f}{\sqrt{2}}, \quad \mathrm{sign}(q) = -\mathrm{sign}(F_{v\eta}) \equiv -\mathrm{sign}(c_F),\tag{85}$$

$$\frac{1}{r_0^2} = 4\pi\left(|q| + \frac{f}{\sqrt{2}}\right)^2 + \Lambda, \quad m = \frac{r_0}{2}\left[1 - \frac{1}{3}\left(\Lambda + 2\pi f^2\right)r_0^2\right],\tag{86}$$

$$\eta_0 = \left[2\pi(f^2 - 2q^2) + \Lambda\right]^{-\frac{1}{2}},\tag{87}$$

$$b_0 = \frac{1}{4}\left(1 - \left(\Lambda + 2\pi f^2\right)r_0^2 + 2r_0\sqrt{\Lambda + 2\pi f^2 - 4\pi q^2}\right),$$

$$T^2 = \frac{b_0}{4\pi}\left[\left(|q| + \frac{f}{\sqrt{2}}\right)^2 + \frac{1}{4\pi}\Lambda\right].\tag{88}$$

The bare cosmological constant must be in the interval:

$$\Lambda \le 0, \quad |\Lambda| < 2\pi(f^2 - 2q^2) \to |q| < \frac{f}{\sqrt{2}},\tag{89}$$

in particular, Λ could be zero.

Again, as in the previous section, relations (85) are of primary importance. They tell us that:

- The "left-most" (76)–(78) and "right-most" (82)–(84) noncompact "universes" become two identical copies of the *electrically neutral* exterior region of Schwarzschild–de Sitter black hole beyond the Schwarzschild horizon carrying a constant vacuum radial electric field with magnitude $|F_{v\eta}| = |F_{0r}| = \frac{f}{\sqrt{2}}$ pointing inbound towards the horizon in one of these "universes" and pointing outbound w.r.t. the horizon in the second "universe." The corresponding electric displacement field $\mathbf{D} = 0$, so there is *no* electric flux there (cf. Eq. (72)–(74)).
- The whole electric flux produced by the two charged *LL-branes* with opposite charges $\pm q$ at the boundaries of the above noncompact "universes" is *confined* within the "tube-like" middle "universe" (79)–(81) where the constant electric field is $|F_{v\eta}| = \frac{f}{\sqrt{2}} + |q|$ with associated nonzero electric displacement field $|\mathbf{D}| = |q|$ (cf. Eqs. (72)–(74)).

The geometry of the charge-confining wormhole is visualized in the Fig. 2.

8. Discussion and Conclusions

In this paper we have studied bulk gravity/nonlinear-gauge-field system self-consistently coupled to one or two charged lightlike branes as matter sources. An important feature of this system is the special form of the nonlinear gauge field sector in (43) previously known to produce QCD-like confining dynamics in flat space–time.[3–7] The main objective here was to search for similar charge confining behavior in curved space–time, where the role of charged objects subject to confinement is played by charged lightlike branes.

We found that charge-confining or charge-"hiding" effects take place within *wormhole* solutions to the coupled gravity/nonlinear-gauge-field/lightlike-brane system (43) with the following special structure:

(i) One of the "universes" comprising the total wormhole space–time manifold must be a compactified "universe" of Levi-Civita–Bertotti–Robinson ("tube-like") type with geometry $\mathcal{M}_2 \times S^2$ where the two-dimensional manifold $\mathcal{M}_2$ possesses at least one horizon.

(ii) The one or two outer "universe(s)" are noncompact spherically symmetric with at least one Schwarzschild-type horizon.

(iii) The matching (gluing together) of the compactified "universe" with the (one of the two) outer noncompact "universe(s)" takes place at a common horizon of both of them, which is automatically occupied by (one of the participating) charged lightlike brane(s) ("horizon straddling" as dictated by worldvolume lightlike brane dynamics).

(iv) Due to the presence of the "square-root" Maxwell term in (43) a nonzero constant vacuum electric field is generated in (any of) the outer noncompact "universe(s)," however, the total flux is *zero* there because of vanishing of
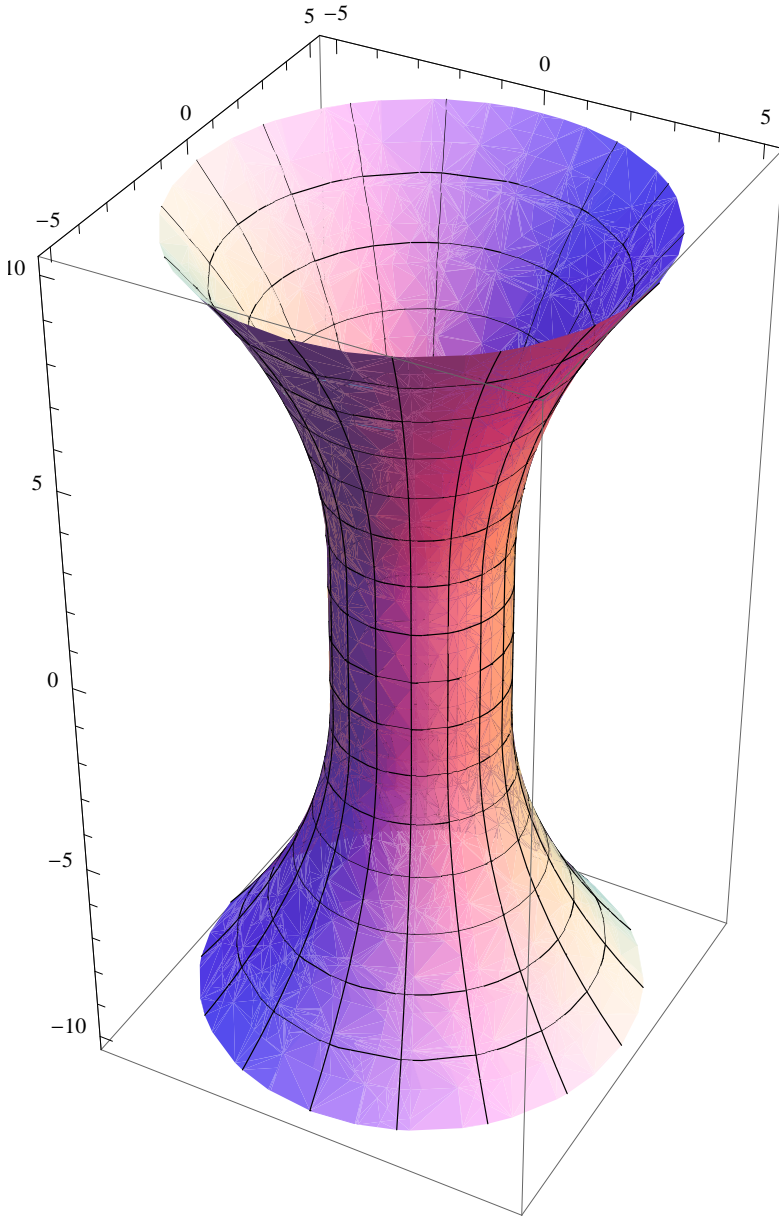
Fig. 2. Shape of $t = $ const and $\theta = \frac{\pi}{2}$ slice of charge-confining wormhole geometry. The whole electric flux is confined within the middle cylindric tube.

the pertinent electric displacement field, so that the charged lightlike brane occupying the "throat" between the noncompact and the compactified "tube-like" "universes" appears as electrically neutral to an external observer in the noncompact "universe."

(v) In the compactified "tube-like" "universe" the charged lightlike brane(s) at the its "border(s)," where it is matched to the noncompact "universe(s)," produce a *nonzero* flux entirely confined within the "tube-like" "universe."

(vi) When only one charged lightlike brane is present, the compactified "tube-like" "universe" with geometries $\mathcal{M}_2 \times S^2$, where $\mathcal{M}_2 = \mathrm{AdS}_2$ or $\mathcal{M}_2 = \mathrm{Rind}_2$, has an infinite extent w.r.t. "radial-like" η-coordinate of $\mathcal{M}_2$ and it absorbs entirely the whole flux produced by the brane at the "border." In this way it hides the charge of the brane from an outside observer in the "neighboring" noncompact "universe."

(vii) When two oppositely charged but otherwise identical lightlike branes are present, the middle "tube-like" "universe" stretching between them has geometry $dS_2 \times S^2$ and has final extent w.r.t. "radial-like" η-coordinate of the dS_2 component. It absorbs entirely the whole flux produced between the branes at its "borders," i.e. the whole flux is *confined* within the finite-extent "tube-like" region without flowing into any of the outside noncompact space–time regions.

It is natural to expect that in a confining theory the gauge field prefers flux-tube configurations, however, the mathematical details of how this happens might be complicated in flat space–time. On the other hand, in the present curved space–time model we obtain the following clear and simple picture:

(a) Due to the presence of lightlike brane(s) as material source(s) of gravity and gauge forces, the very special lightlike brane worldvolume dynamics *triggers* one or more transitions between noncompact and compactified "tube-like" space–time regions in the form of special wormhole configurations with the lightlike brane(s) sitting at the "throat(s)."

(b) Again the special lightlike brane worldvolume dynamics in combination with the special properties of the additional "square-root" Maxwell term in the nonlinear gauge field action cause the whole flux generated by the charged branes to be entirely confined within the compactified "tube-like" region.

As a final remark, returning to the nonlinear gauge field equations (5) we see that there exists a more general *vacuum* solution of the latter *without* the assumption of staticity and spherical symmetry:

$$F^2 \equiv F_{\kappa\lambda}F_{\mu\nu}G^{\kappa\mu}G^{\lambda\nu} = -f^2 = \mathrm{const}. \qquad (90)$$

The latter automatically produces via Eq. (4) an effective positive cosmological constant:

$$T^{(F)}_{\mu\nu} = -\frac{f^2}{4}G_{\mu\nu}, \quad \text{i.e.} \quad \Lambda_{\mathrm{eff}} = 2\pi f^2. \qquad (91)$$

This reduces the gravity/gauge-field equations of motion (3), (5) to the vacuum Einstein equations (with effective cosmological constant):

$$R_{\mu\nu} - \frac{1}{2}G_{\mu\nu}R + (\Lambda + 2\pi f^2)G_{\mu\nu} = 0 \qquad (92)$$

supplemented with the constraint Eq. (90). Thus, assuming absence of magnetic field ($F_{mn} = 0$), i.e. $F^2 = 2E_m E_n G^{mn} G^{00}$, $E_m \equiv F_{0m}$ ($m, n = 1, 2, 3$), we obtain the above described electrically neutral Schwarzschild–(anti)-de Sitter or purely Schwarzschild solutions with a constant vacuum electric field (10), which according to (90) has constant magnitude:

$$|\mathbf{E}| \equiv \sqrt{-\frac{1}{2}F^2} = \frac{f}{\sqrt{2}}\,, \tag{93}$$

but it may point in arbitrary direction. In this vacuum with disordered constant-magnitude electric field it will not be able to pass energy to a test charged particle, which instead will undergo a kind of Brownian motion, therefore *no* Schwinger pair-creation mechanism will take place.

Acknowledgments

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